

RESOLVIENDO PROBLEMAS DE MATEMÁTICA

RESOLUCIÓN DE LOS PROBLEMAS PROPUESTOS

PROBLEMA 151 (100118)

Resuélvase la integral triple

$$\int_0^1 \int_0^1 \int_0^1 \frac{dz}{\sqrt{x+y+z+1}}$$

RESOLUCIÓN:

$$\int_0^1 \frac{dz}{\sqrt{x+y+z+1}} = 2(z+(x+y+1))^{\frac{1}{2}} \Big|_0^1 = 2(\sqrt{x+y+2} - \sqrt{x+y+1})$$

$$\begin{aligned} \int_0^1 2(\sqrt{x+y+2} - \sqrt{x+y+1}) dy &= 2 \cdot \frac{2}{3} (y+(x+2))^{\frac{3}{2}} - 2 \cdot \frac{2}{3} (y+(x+1))^{\frac{3}{2}} \Big|_0^1 = \\ &= \frac{4}{3} \left[(x+3)^{\frac{3}{2}} - (x+2)^{\frac{3}{2}} - (x+2)^{\frac{3}{2}} + (x+1)^{\frac{3}{2}} \right] = \frac{4}{3} \left[(x+3)^{\frac{3}{2}} - 2(x+2)^{\frac{3}{2}} + (x+1)^{\frac{3}{2}} \right] \end{aligned}$$

$$\begin{aligned} \int_0^1 \frac{4}{3} \left[(x+3)^{\frac{3}{2}} - 2(x+2)^{\frac{3}{2}} + (x+1)^{\frac{3}{2}} \right] dx &= \frac{8}{15} \left[(x+3)^{\frac{5}{2}} - 2(x+2)^{\frac{5}{2}} + (x+1)^{\frac{5}{2}} \right] \Big|_0^1 = \\ &= \frac{8}{15} \left[(4^{\frac{5}{2}} - 2 \cdot 3^{\frac{5}{2}} + 2^{\frac{5}{2}}) - (3^{\frac{5}{2}} - 2 \cdot 2^{\frac{5}{2}} + 1) \right] = \frac{8}{15} \left[2^5 - 3^3 \cdot 3^{\frac{1}{2}} + 3 \cdot 2^2 \cdot 2^{\frac{1}{2}} - 1 \right] = \\ &= \frac{8}{15} [31 - 27\sqrt{3} + 12\sqrt{2}] \end{aligned}$$